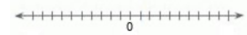
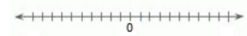
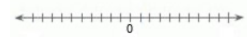
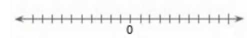
Solving Absolute Value Inequalities

Absolute Value Inequalities

$|x| \le 2$ is an example of an **absolute value inequality**.

Basic Absolute Value **Inequality** $(k > 0)$

Absolute Value Inequality	Equivalent Inequality	Solution Set	Graph (for you to do)
$\| x \| > k$	$x > k$ or $x < -k$	$(-\infty, -k) \cup (k, \infty)$	
$\| x \| \ge k$	$x \ge k$ or $x \le -k$	$(-\infty, -k] \cup [k, \infty)$	
$\| x \| < k$	$-k < x < k$	$(-k, k)$	
$\| x \| \le k$	$-k \le x \le k$	$[-k, k]$	

$|x| \le 2$ is equivalent to $-2 \le x \le 2$ The solution set is $[-2, 2]$

Compound Inequalities

The Equivalent Inequalities shown in the chart above are examples of **compound inequalities**.

If one joins two simple inequalities with the connective "or" or the connective "and", one gets a **compound inequality**.

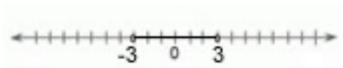
$x < -8$ or $x > 8$ A compound inequality using the connective "OR" is true if one **or** the other **or** both of the simple inequalities are true. It is false only if both simple inequalities are false. The solution set to an "OR" compound inequality consists of all numbers that satisfy at least one of the simple inequalities (the union of the numbers).
$(-\infty, -8) \cup (8, \infty)$

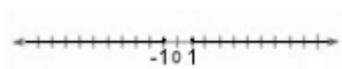
$x > -8$ and $x < 8$ (often written in the form $-8 < x < 8$)
A compound inequality using the connective "AND" is true if and only if **both** simple inequalities are true. The solution set to an "AND" compound inequality consists of all numbers that satisfy both simple inequalities (the intersection of the numbers).
$(-8, 8)$

When working in word problems, the word **between** translates into a compound inequality using strict inequality symbols < or >.

Words **from** and **to** translates into weak inequality symbols $\le$ or $\ge$.

Use compound inequalities to solve these Basic Absolute Value Inequalities.

$\lvert w \rvert < 3$  $-3 < w$ and $w < 3$ $-3 < w < 3$ $(-3, 3)$	First, isolate the absolute value. (It is.) Next, draw a number line marking 0, the value and its opposite. That is, mark the 3 and -3. Since this is a less than, indicate the area of the line between 0 and 3 that is represented by the statement: $w < 3$. Now, indicate the opposite area … the area between 0 and -3. This represents an AND. Rewrite the problem into two parts and solve.

$\lvert w - 5 \rvert \geq 1$  $w - 5 \leq -1$ or $w - 5 \geq 1$ $w - 5 + 5 \leq -1 + 5$ or $w - 5 + 5 \geq 1 + 5$ $w \leq 4$ or $w \geq 6$ $(-\infty, 4] \cup [6, \infty)$	First, isolate the absolute value. Next, draw a number line marking 0, the value and its opposite. That is, mark the 1 and -1. Since this is a greater than, indicate the area greater than 1 that is represented by w-5. Now, indicate the opposite area … the area less than -1. This represents an OR. Rewrite the problem into two parts and solve. The final answer cannot be combined. It is written as the union of two intervals.

TRY: $\lvert 3x \rvert < 15$

$4 \le |x| - 6$

$-3|6 - x| \ge -3$

$|6 - x| \ge 0$