Completing the Square

Think about $x^2 + 8x + 16$.

This can be written $(x+4)^2$.

The expression $x^2 + 8x + 16$ is a perfect square trinomial.

Perfect square trinomials: $a^2 - 2ab + b^2 = (a-b)^2$ and $a^2 + 2ab + b^2 = (a+b)^2$

Necessary Conditions for a Perfect Square Trinomial

4. The first term must have a positive coefficient and be a perfect square, a^2.
5. The last term must have a positive coefficient and be a perfect square, b^2.
6. The middle term must be twice the product of the bases of the first and last terms, $2ab$ or $-2ab$.

Is $x^2 + 16x + 64$ a Perfect Square Trinomial?

YES – since it can be factored: $(x+8)^2$

Perfect square trinomials are created when a binomial is squared.

Think about the following: $x^2 + 6x + ?$

What number could be used for the $?$ to create a perfect square trinomial?

$$x^2 + 6x + \underline{9} \quad \text{or} \quad (x+3)^2$$

Complete the following perfect square trinomial: $(x + \underset{5}{\_})^2 = x^2 + 10x + \underset{25}{?}$

Now think of the process in reverse. What values make these into perfect square trinomials?

$$x^2 + 14x + \underset{49}{\_\_} = (x + \underset{7}{\_\_})^2 \qquad x^2 - 12x + \underset{36}{\_\_} = (x - \underset{6}{\_\_})^2$$

What process will always work for $ax^2+bx+c=0$ when a=1?

x^2+14x+ ___ $(x+$ ___ $)^2$	$x^2+14x+\underline{49}$ $\downarrow \quad \downarrow \tfrac{1}{2}$ $(x+\underline{7})^2$	1. Be sure the sign of the b term in the trinomial is the same as the sign in the binomial. 2. Take ½ of the coefficient of the b term of the trinomial and place it in the blank of the binomial. 3. Square the last term of the binomial and use the result for the c term of the trinomial.
$x^2-12x+\underline{36}$ $\downarrow \quad \downarrow \tfrac{1}{2}$ $(x-\underline{6})^2$	$x^2-5x+\underline{\tfrac{25}{4}}$ $\downarrow \quad \downarrow$ $(x-\underline{\tfrac{5}{2}})^2$	$x^2+\frac{6}{7}x+\underline{\tfrac{9}{49}}$ $\downarrow \quad \downarrow (\tfrac{1}{2})$ $(x+\underline{\tfrac{3}{7}})^2$ Even if the b is a fraction, still take ½ of it.

<u>Completing the Square</u>

The process of transforming the quadratic equation $ax^2+bx+c=0$ into the form $(x+q)^2=k$ where q and k are constants is called **completing the square**.

If one knows the first and second terms, one can find the last term to make a perfect square trinomial. It is *the square of* ½ of the coefficient of the middle term.

TRY:

$m^2 + 14m +$ ___ $\qquad$ $w^2 - 5w +$ ___ $\qquad$ $p^2 + \frac{6}{5}p +$ ___

$(m +$ ___ $)^2$ $\qquad$ $(w -$ ___ $)^2$ $\qquad$ $(p +$ ___ $)^2$

Do you see how easily these perfect square trinomials can be factored? TRY THESE.

$y^2-10y+25$ $\qquad$ $w^2+w+\frac{1}{4}$ $\qquad$ $m^2-\frac{6}{5}m+\frac{9}{25}$

$(y-5)^2$ $\qquad$ $\left(w+\frac{1}{2}\right)^2$ $\qquad$ $\left(m-\frac{3}{5}\right)^2$