Square Root Property

What number times itself is 9? 3 Is there another number that also works? -3

So in looking at $x^2 = 9$, either 3 or -3, when substituted in place of x, would make the statement true. In solving quadratic equations, we now need to consider both the positive and the negative results.

Square Root Property (sometimes called the Even Root Property)

When *n* is a positive even integer,

if $k > 0$, then $x^n = k$ is equivalent to $x = \pm\sqrt[n]{k}$. $x^2 = 4$ is equivalent to $x = \pm 2$

if $k = 0$, then $x^n = k$ is equivalent to 0. $x^2 = 0$ is equivalent to $x = 0$

if $k < 0$, then $x^n = k$ has no real solution. $x^2 = -4$ has no real solution

Examples:

Solve for the unknown using the Square Root Property

$$x^2 = \frac{9}{4}$$
$$\sqrt{x^2} = \sqrt{\frac{9}{4}}$$
$$x = \pm\frac{3}{2}$$
$$\left\{-\frac{3}{2}, \frac{3}{2}\right\}$$

$$a^2 = 32$$
$$\sqrt{a^2} = \sqrt{32}$$
$$a = \pm 4\sqrt{2}$$
$$\{-4\sqrt{2}, 4\sqrt{2}\}$$

$$x^2 = 25$$
$$\sqrt{x^2} = \sqrt{25}$$
$$x = \pm 5$$
$$\{-5, 5\}$$

TRY:

$$x^2 = \frac{16}{25}$$

$$x^2 = 98$$

Would the same process work to solve for (x-4):

$$(x-4)^2 = 25$$

$$\sqrt{(x-4)^2} = \sqrt{25}$$

$$(x-4) = \pm 5$$

Now solve for x. $x-4=5$ or $x-4=-5$

$x=9$ or $x=-1$

$\{-1, 9\}$

The process of using the square root property to solve certain forms of quadratic equations is called **extracting the roots**.

$$(a-3)^2 = 8$$

$$\sqrt{(a-3)^2} = \sqrt{8}$$

$$a-3 = \pm 2\sqrt{2}$$

$a-3=+2\sqrt{2}$ or $a-3=-2\sqrt{2}$

$a=3+2\sqrt{2}$ or $a=3-2\sqrt{2}$

$\{3-2\sqrt{2}, 3+2\sqrt{2}\}$

$$(2m-7)^2 = 12$$

$$\sqrt{(2m-7)^2} = \sqrt{12}$$

Remember: $\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$

$$2m-7 = \pm 2\sqrt{3}$$

$$2m = 7 \pm 2\sqrt{3}$$

$$m = \frac{7 \pm 2\sqrt{3}}{2}$$

$$\left\{\frac{7-2\sqrt{3}}{2}, \frac{7+2\sqrt{3}}{2}\right\}$$

Do not be tempted to remove the 2's in this answer. In order to reduce by 2, a factor of 2 must be able to be factored out from each of the two terms in the numerator. There is not a factor of 2 in the number 7, so one cannot factor out a 2. Therefore, one cannot reduce the fraction by 2.

TRY:

$$(x-5)^2 = 9 \qquad\qquad (2x+7)^2 = 15$$

Using the Square Root property with Complex Numbers

$x^2 = -49$ $\sqrt{x^2} = \sqrt{-49}$ $x = \pm\sqrt{-49}$ $x = \pm\sqrt{-1 \cdot 49}$ $x = \pm 7i$ The solution set is $\{-7i, 7i\}$	$(k+5)^2 + 13 = 4$ $(k+5)^2 = 4 - 13$ $(k+5)^2 = -9$ $\sqrt{(k+5)^2} = \sqrt{-9}$ $k+5 = \pm 3i$ $k = -5 \pm 3i$ The solution set is $\{-5-3i, -5+3i\}$	$(5m-7)^2 + 20 = 0$ $(5m-7)^2 = -20$ $\sqrt{(5m-7)^2} = \sqrt{-20}$ $5m - 7 = \pm\sqrt{-20}$ $5m - 7 = \pm i\sqrt{4 \cdot 5}$ $5m - 7 = \pm 2i\sqrt{5}$ $5m = 7 \pm 2i\sqrt{5}$ $m = \dfrac{7 \pm 2i\sqrt{5}}{5}$ The solution set is $\left\{\dfrac{7-2i\sqrt{5}}{5}, \dfrac{7+2i\sqrt{5}}{5}\right\}$

TRY:

$$x^2 = -36$$

$$(k-3)^2 + 12 = 4$$

$$(2m+3)^2 + 14 = 6$$