# Rational Equations: Proportions

Proportions

An equation that expresses the equality of two rational expressions is called a **proportion**.

**Extremes–Means Property.**     If $\dfrac{a}{b} = \dfrac{c}{d}$ , then $ad = bc$ . This equation is a proportion.

In it, $a$ and $d$ are called the **extremes**; $b$ and $c$ are called the **means**.

$\dfrac{5}{x} = \dfrac{7}{9}$   can be written   $5 \cdot 9 = 7x$        $45 = 7x$        $\dfrac{45}{7} = x$     Solution set:  $\left\{ \dfrac{45}{7} \right\}$

Examples:

$\dfrac{3}{x-2} = \dfrac{x+2}{7}$ ➜ $3(7) = (x+2)(x-2)$ ➜

$21 = x^2 - 4$ ➜ $0 = x^2 - 25$ ➜ $0 = (x+5)(x-5)$ ➜

$x + 5 = 0$ or $x - 5 = 0$ ➜ $x = -5$ or $x = 5$

Are -5 and 5 part of the Domain? Yes.   Solution: $\{-5,5\}$

- If an equation involves only two rational expressions, make sure one is on each side of the equal sign and use the **Extremes-Means** technique (that is, cross multiply to form an equation to solve).

$\dfrac{3}{2x-5} + \dfrac{2}{2x+3} = 0$   Move one fraction to the other side (Don't forget the sign change.)

$\dfrac{3}{2x-5} = \dfrac{-2}{2x+3}$   Use the Means-Extremes technique to solve.

$3(2x+3) = -2(2x-5)$ ➜    $6x + 9 = -4x + 10$  ➜ $6x + 4x = 10 - 9$

Combine terms   $10x = 1$ ➜ $\dfrac{10x}{10} = \dfrac{1}{10}$ ➜ $x = \dfrac{1}{10}$

Check the domain. The possible solution is ok. Solution: $\left\{ \dfrac{1}{10} \right\}$

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- If an equation involves three rational expressions AND it is easy to combine two in order to make the equation proportion, do so. Then use the **Extremes-Means** technique to solve.

$$\frac{5}{2w+6} - \frac{1}{w+3} = \frac{1}{w-1}$$

If the middle rational expression were multiplied by $\frac{2}{2}$, it would have the same denominator as the first fraction and could easily be combined with the first one.

This would result in: $\quad \dfrac{5}{2w+6} - \dfrac{2}{2w+6} = \dfrac{1}{w-1}$ ➜

$$\frac{3}{2w+6} = \frac{1}{w-1} \ ➜ \ 3(w-1) = 2w+6 \ ➜$$

$$3w-3 = 2w+6 \ ➜ \ w = 9$$

Check the domain or the ORIGINAL equation. The possible solution is ok. Solution: $\{9\}$

This method may be more time consuming. It is offered only as an alternative.

TRY:
Find the solution set to each equation.

$$\frac{3}{8} = \frac{5}{x} \qquad\qquad\qquad\qquad \frac{a-5}{a+6} = \frac{a-7}{a+8}$$

$$\frac{7}{3x-9} - \frac{1}{x-3} = \frac{4}{9}$$