Lesson 17: Solving Rational Equations

Rational Equations

Rational Equation
An equation that contains a rational expression is called a **rational equation**.

Know the difference!

Rational Expression: $\frac{3}{x}-\frac{1}{5}$	Rational Equation: $\frac{3}{x}-\frac{1}{5}=0$
One **evaluates** the **expression**.	One **solves** the **equation**.
When working with a Rational Expression, one **maintains** the denominator by converting every term of the equation to an equivalent fraction of the LCD.	When working with a Rational Equation, one **eliminates** the denominator by multiplying every term of the equation by the LCD.
The **result** is a simplified expression. $\frac{15-x}{5x}$	The **solution**, x=15, is shown in a solution set. { 15 }

Solving a Rational EQUATION

1. Find the LCD of **all** rational expressions in the equation.

2. Eliminate the denominators by multiplying each term of **both** sides of the equation by the LCD of the rational expressions in the equation.

3. Use the procedure for solving linear equations to solve the resulting equation.

4. **Check** the solution to see that it makes the rational equation true. Take extra care to check your solution when a variable appears in the denominator. Because equations involving rational expressions have variables in denominators, a solution (root) to the equation might cause a 0 to appear in the denominator making the rational expression undefined. In this case, the solution (root) does not satisfy the original equation, and so it is called an **extraneous root**.

Example:

$$\frac{12}{3x^2+12x}=1-\frac{1}{x+4}$$

- If the equation involves three or more rational expressions being added or subtracted, first check to see if any fraction can be reduced, and do so.
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FACTOR first $\frac{3\bullet 4}{3x(x+4)}=1-\frac{1}{x+4}$ then reduce

- Find the LCD (lowest common denominator).

$\frac{4}{x(x+4)}=1-\frac{1}{x+4}$ The LCD is $x(x+4)$.

- Multiply **every** term by the LCD: $\frac{4}{x(x+4)}\cdot(x)(x+4)=1\cdot(x)(x+4)-\frac{1}{x+4}\cdot(x)(x+4)$

- If you have the correct LCD, all denominators will cancel out.

$$4=1(x)(x+4)-1(x)$$

- Continue to solve.

$4=x^2+4x-x$ Combine like terms and move everything to one side.

$0=x^2+3x-4$ Factor: $0=(x+4)(x-1)$ Either $0=x+4$ or $0=x-1$

that is $x=-4$ or $x=1$; **Possible** Solutions: -4 or 1

NOW CHECK to see if -4 and 1 can be in the domain.

-4 makes the denominator of the first fraction undefined;
1 is okay.

Solution: $\{1\}$

Example:

$$\frac{5}{6x}-\frac{1}{8x}=\frac{17}{24}$$

Find the LCD:

$\frac{5}{6x}-\frac{1}{8x}=\frac{17}{24}$	6x	2 3 x
	8x	2 x 2 2
	LCD:	2 3 x 2 2 = 24x

$$\frac{5}{6x}-\frac{1}{8x}=\frac{17}{24} \Rightarrow \frac{5\cdot 24x}{6x}-\frac{1\cdot 24x}{8x}=\frac{17\cdot 24x}{24} \Rightarrow 5\cdot 4-1\cdot 3=17\cdot x \Rightarrow 20-3=17x$$

$$\Rightarrow 17=17x \Rightarrow \frac{17}{17}=\frac{17x}{17} \Rightarrow 1=x$$

Is 1 part of the Domain? Yes.

Therefore the solution is: $\{1\}$

Example:

$$\frac{x-2}{x-6}-\frac{4}{x}=\frac{24}{x^2-6x}$$

Find the LCD:

$\frac{x-2}{x-6}-\frac{4}{x}=\frac{24}{x^2-6x}$	$x-6$	$(x-6)$
	x	x
	x^2-6x	$(x-6)\quad x$
	LCD:	$(x-6)\quad x = x\,(x-6)$

Be sure the denominators are factored first.

$$\frac{x-2}{x-6}-\frac{4}{x}=\frac{24}{x(x-6)} \Rightarrow \frac{(x-2)(x)(x-6)}{x-6}-\frac{4(x)(x-6)}{x}=\frac{24(x)(x-6)}{(x)(x-6)} \Rightarrow$$

$$(x-2)(x)-4(x-6)=24 \Rightarrow x^2-2x-4x+24=24 \Rightarrow x^2-6x=0$$

$x(x-6)=0 \Rightarrow x=0$ or $(x-6)=0 \Rightarrow x=0$ or $x=6$

Are 0 and 6 part of the Domain? Will either of them make a denominator in the equation 0?

Both values are excluded from the domain. Therefore, there is no solution. $\varnothing$

TRY:

Find the solution set to each equation.

$$\frac{3}{x}+\frac{1}{5}=\frac{1}{2}$$

$$\frac{5}{x-1}+\frac{1}{2x}=\frac{1}{x}$$

$$\frac{x-3}{x+2}=3-\frac{1-2x}{x+2}$$